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On natural convection in enclosures filled with fluid-saturated porous media including viscous dissipation

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Abstract

Care needs to be taken when considering the viscous dissipation in the energy conservation formulation of the natural convection problem in fluid-saturated porous media. The unique energy formulation compatible with the First Law of Thermodynamics informs us that if the viscous dissipation term is taken into account, also the work of pressure forces term needs to be taken into account. In integral terms, the work of pressure forces must equal the energy dissipated by viscous effects, and the net energy generation in the overall domain must be zero. If only the (positive) viscous dissipation term is considered in the energy conservation equation, the domain behaves as a heat multiplier, with an heat output greater than the heat input. Only the energy formulation consistent with the First Law of Thermodynamics leads to the correct flow and temperature fields, as well as of the heat transfer parameters characterizing the involved porous device. Attention is given to the natural convection problem in a square enclosure filled with a fluid-saturated porous medium, using the Darcy Law to describe the fluid flow, but the main ideas and conclusions apply equally for any general natural or mixed convection heat transfer problem. It is also analyzed the validity of the Oberbeck–Boussinesq approximation when applied to natural convection problems in fluid-saturated porous media.

1. Introduction

There is an increasing interest in the study of natural convection in fluid-saturated porous media, as proved by the explosive growth in the literature on the subject, and also an increasing interest in the consideration of the viscous dissipation effects on the flow and temperature fields, as well as on the heat transfer performance of the involved devices. From an order of magnitude analysis it can be concluded that the viscous dissipation can be neglected in many situations of practical interest, both for domains filled with a clear fluid or for domains filled with fluid-saturated porous media. This is, however, a subject that attracts many research workers and, in particular, special attention is being devoted to the natural convection in enclosures filled with a fluid-saturated porous medium including the viscous dissipation effects. In this work, it is studied the natural convection in a square enclosure, but the main results and conclusions apply to any natural or mixed convection problem in fluid-saturated porous media. The corresponding problem, relative to a square enclosure filled with a clear fluid, has been studied recently by Costa [1], and the interest on this problem can be assessed by the references cited herein and also by the very recent works of Pons and Le-Queré [2–4].

Going on to the literature, one can find many recent works concerning the natural convection in fluid-saturated porous media, including viscous dissipation effects. Examples of works considering the Darcy Law to describe the fluid flow are these of Nakayama and Pop [5], Magyari and Keller [6], Rees et al. [7], Saeid and Pop [8] and Rees [9]. In the work of Al-Hadhrami et al. [10] it is considered the Brinkman extension of the Darcy Law, and a quadratic drag term on the momentum equation is considered in the works of Murthy and Singh [11], Murthy [12], Tashtoush [13] and Magyari et al. [14]. The book by Nield and Bejan

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Nomenclature

c_P	constant pressure specific heat
Da	Darcy number
E	total energy
Ec	Eckert number
g	gravitational acceleration
$\mathbf{g}$	gravitational acceleration vector
H	height
$\hat{h}$	specific enthalpy
k	thermal conductivity
K	permeability
m	mass
Nu	Nusselt number
p	pressure
Pr	Prandtl number
$\mathbf{q}$	heat flux vector
$\dot{Q}$	heat flow
Ra	Darcy-modified Rayleigh number
$\dot{S}$	entropy flow
$\dot{S}'''$	volumetric rate of entropy generation
t	time
T	temperature
u, v	Cartesian velocity components
$\hat{u}$	specific internal energy
V	volume
$\mathbf{v}$	surface velocity vector
$\mathbf{V}$	intrinsic velocity vector
$\dot{W}$	mechanical power
x, y	Cartesian co-ordinates

Greek symbols

a	thermal diffusivity
β	volumetric expansion coefficient
ΔT	temperature difference
ε	porosity
μ	dynamic viscosity
ν	kinematic viscosity
ρ	density
τ	temperature ratio
ψ	(conservative) streamfunction

Subscripts

C	cold (lower temperature) value
CD	conduction
D	viscous dissipation
f	fluid
gen	generation
H	hot (higher temperature) value
m	combined
0	reference value
s	solid
*	dimensionless

[15] gives a very good description about the relevance of the subject of heat transfer in porous media, and about the models used to take into account the different effects on the natural convection in fluid-saturated porous media. Nield [16] gives an explanation why the quadratic drag term on the momentum equation (which does not contain the viscosity in an explicit way) must be taken into account as a dissipation term. A study of the entropy generation associated with the natural convection heat transfer problem in an inclined square enclosure filled with a fluid-saturated porous medium was conducted by Baytas [17]. In this work, the viscous dissipation term is not taken into account in the energy conservation equation, but it is taken into account in the entropy generation equation.

In all the previously referred works, concerning natural convection in fluid-saturated porous media, with the exception of the book by Nield and Bejan [15], no reference is made to the work of pressure forces term when the viscous dissipation term is taken into account in the energy conservation equation. In fact, as it is shown in the present work, both terms need to be considered in order to have the unique energy conservation formulation that is consistent with the First Law of Thermodynamics.

If we consider an enclosure filled with a fluid-saturated porous medium, like the one presented in Fig. 1, where steady natural convection takes place, and the viscous dissipation term is considered in the energy conservation equation, a net heat generation takes place in the enclosure and heat leaving the enclosure is greater than that entering the enclosure. Such an enclosure behaves like a heat multiplier, which is inconsistent regarding the First Law of Thermodynamics. It must be noted that viscous dissipation is due to fluid motion, and that fluid motion is not forced

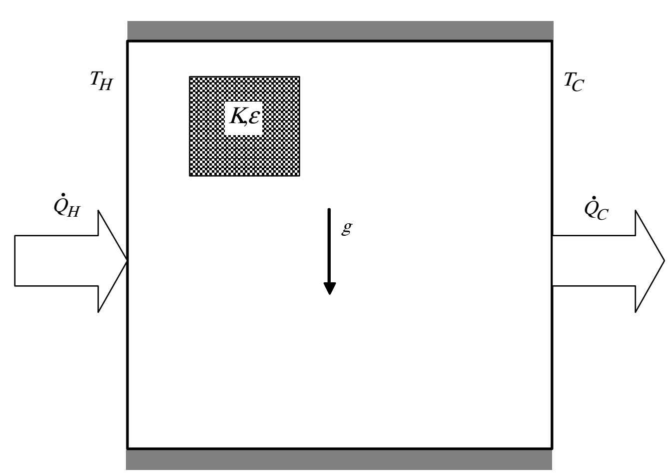

Fig. 1. The natural convection problem in a differentially heated square enclosure filled with a fluid-saturated porous medium.

(originated by an external mechanical action) but it is the result of the expansion–contraction cycle experienced by the fluid flowing in the enclosure. Fluid motion results from the use of some of the heat entering the enclosure to increase the temperature of the fluid and thus to decrease its density. Fluid motion is associated with the work of pressure forces involved in the expansion–contraction cycle experienced by the fluid, and the unique energy conservation formulation consistent with the First Law of Thermodynamics is that considering, simultaneously, the viscous dissipation and the work of pressure forces in the energy conservation equation.

If only the viscous dissipation term is considered in the energy conservation equation, results are not valid regarding the First Law of Thermodynamics, and the flow and temperature fields are also very different from the ones obtained using the consistent energy conservation formulation. Also very different are the results obtained to express the thermal performance of the fluid-saturated porous devices where natural convection takes place. Additionally, it is obtained a very useful criterion to assess if the energy formulation of the problem is correct or not, and if the Oberbeck–Boussinesq approximation can be used when dealing with natural convection in fluid-saturated porous media.

2. Thermodynamics and analysis of the problem

Consider the closed square enclosure filled with an isotropic and homogeneous porous medium of permeability K, as presented in Fig. 1, whose left and right vertical walls are maintained at constant temperatures T_H and T_C, respectively. The porous medium is saturated with a fluid that expands when its temperature increases ($\beta > 0$), even if some particular fluids and/or conditions can be pointed out for which $\beta < 0$. The upper and lower horizontal walls are assumed to be perfectly insulated.

Close to the hot isothermal wall the fluid is heated and expands, thus giving rise to an ascending motion. The fluid changes its direction when reaching the neighborhood of the top horizontal wall, proceeds nearly in the horizontal direction from left to right, and changes direction when reaching the neighborhood of the cold vertical wall. As the fluid releases heat there, its temperature decreases, it becomes denser and sinks down. Close to the lower horizontal wall the flow is essentially horizontal. In the combined medium (solid porous matrix and saturating fluid) heat is transferred by conduction, and it is assumed that local thermal equilibrium exists. A closed loop is established for the fluid flow, and the combined conduction–convection action transfers heat from the hot wall to the cold wall.

An equilibrium situation is reached for which the temperature difference gives rise to the fluid motion and the viscous dissipation action brakes the fluid flow. From a thermodynamic viewpoint this situation can be seen in the following way: the temperature difference $T_H - T_C$ could move a thermal engine if it were present, connected to a fan (fluid current due to the expansion–contraction cycle), but a brake limits the operation of that thermal engine (viscous dissipation acts as a brake for the fluid current). When equilibrium is reached, power obtained from the expansion–contraction cycle is viscously dissipated as heat. This situation was discussed before in [1] for the enclosure filled with a clear fluid. The interpretation of the natural convection in enclosures as a heat engine was explored in [1,18,19] for an enclosure filled with a clear fluid, the main aspects of such an analysis applying also for the enclosure filled with a fluid-saturated porous medium.

First Law of Thermodynamics applied for the overall enclosure (a closed thermodynamic system) gives $\mathrm{d}E/\mathrm{d}t = (\dot{Q}_H + \dot{Q}_C) + \dot{W}$ [20], where E is the total energy, $E = m[\hat{u} + (1/2)m|\mathbf{V}|^2 + gy]$, and $\dot{Q}_H$, $\dot{Q}_C$ and $\dot{W}$ are taken as positive when entering the thermodynamic system and negative otherwise. If the system operates in steady-state conditions then $\mathrm{d}E/\mathrm{d}t = 0$. Additionally, as there is no any rotating shaft or other mechanical device through which the enclosure exchanges mechanical work with its neighborings then $\dot{W} = 0$. In this way, for the overall enclosure $\dot{Q}_H(>0) + \dot{Q}_C(<0) = 0$, that is,

$$\dot{Q} = |\dot{Q}_C| = |\dot{Q}_H| \tag{1}$$

This result is independent of the medium that fills the enclosure, and it has been explored before for the enclosure filed with a clear fluid [1].

The result given by Eq. (1) seems to be a strange result, as viscous dissipation would act like an heat source in the enclosure thus leading to $|\dot{Q}_C| > |\dot{Q}_H|$. In fact, care needs to be taken when analyzing the present problem, as the fluid flow that gives rise to the viscous dissipation is not imposed by any external means (no forced flow), but it results from the thermal levels imposed to the enclosure walls, through the expansion–contraction cycle (work of pressure forces) experienced by the fluid. In this way, viscous dissipation can only be considered if the work of pressure forces is also considered in the energy conservation equation. Viscous dissipation acts like a heat source and the work of pressure forces acts like a heat sink, and globally, over the overall enclosure, their absolute values are equal, even if locally they can be different. From the foregoing arguments, fluid flow is a result of the heat flow crossing the enclosure, as well as the viscous dissipation. In the equilibrium steady state situation no heat is gained or lost by the enclosure, and Eq. (1) prevails, the unique result consistent with the First Law of Thermodynamics. Only when a heat source of different nature than viscous dissipation or the work of pressure forces is present, such as an exothermic chemical reaction or an electrical resistance, does $|\dot{Q}_C| > |\dot{Q}_H|$.

The result given by Eq. (1) can be used advantageously to obtain the overall entropy generation rate in the enclosure as

$$\dot{S}_{\mathrm{gen}} = \dot{Q}\left(\frac{1}{T_{\mathrm{C}}} - \frac{1}{T_{\mathrm{H}}}\right) \tag{2}$$

a result that is independent of the medium that fills the enclosure. However, it is to be noted that the numerical value of $\dot{Q}$ depends on the medium that fills the enclosure. Looking in detail at the mechanisms of entropy generation in the enclosure, it is

$$\dot{S}_{\mathrm{gen}} = (\dot{S}_{\mathrm{gen}})_{\mathrm{CD}} + (\dot{S}_{\mathrm{gen}})_{\mathrm{D}} \tag{3}$$

where $(\dot{S}_{\mathrm{gen}})_{\mathrm{CD}}$ is the rate of entropy generation due to heat conduction trough the combined medium (fluid and porous matrix) and $(\dot{S}_{\mathrm{gen}})_{\mathrm{D}}$ is the rate of entropy generation due to viscous dissipation. Each of these terms can be evaluated alone [21], but the overall rate of entropy generation in the domain can be evaluated from an overall entropy balance for the enclosure, as given by Eq. (2).

3. Physical modeling

3.1. Momentum equation

For the enclosure under analysis, for which the fluid flow is described through the Darcy Law, velocity is expressed as

$$\mathbf{v} = -\frac{K}{\mu}(\nabla p - \rho\mathbf{g}) \tag{4}$$

where $\mathbf{v}$ is the superficial velocity (or the Darcy velocity), averaged over the total cross section of a representative element of the porous medium. However, the term ∇p refers to the volume averaged pressure of the fluid, over the volume of fluid (an intrinsic value), and $-\rho\mathbf{g}$ is the force applied to the fluid, by unit of volume of fluid, that is, it is also an intrinsic term. Intrinsic term is a term referred only to the volume of fluid contained in a given volume of fluid-saturated porous medium [15]. Intrinsic velocity $\mathbf{V}$ and the Darcy velocity $\mathbf{v}$ are related through the Dupuit–Forchheimer relationship according to $\mathbf{v} = \varepsilon\mathbf{V}$ [15]. Assuming that gravity acts downwards in the vertical direction, components of velocity are obtained explicitly as

$$u = -\frac{K}{\mu}\frac{\partial p}{\partial x}; \quad v = -\frac{K}{\mu}\left(\frac{\partial p}{\partial y} + \rho g\right) \tag{5}$$

The mass conservation equation for steady flow reads as

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \tag{6}$$

where u and v are the Cartesian components of $\mathbf{v}$, and the (conservative) streamfunction ψ can be defined through its first-order derivatives as

$$\frac{\partial \psi}{\partial y} \equiv \rho u; \quad -\frac{\partial \psi}{\partial x} \equiv \rho v \tag{7}$$

From the foregoing equations, assuming that pressure is a continuous function to its second-order derivatives, a Poisson equation can be obtained from which the streamfunction field can be evaluated, in the form:

$$0 = \frac{\partial}{\partial x}\left(\frac{\mu}{K\rho}\frac{\partial \psi}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\mu}{K\rho}\frac{\partial \psi}{\partial y}\right) - \frac{\partial}{\partial x}(\rho g) \tag{8}$$

For the problem under analysis temperature is made dimensionless as

$$T_* \equiv (T - T_0)/\Delta T \tag{9}$$

where $T_0 = T_{\mathrm{C}}$ is the minimum temperature value in the enclosure, and ΔT is the maximum temperature difference in the domain, $\Delta T = (T_{\mathrm{H}} - T_{\mathrm{C}})$. In this way, a dimensional temperature can be obtained from the dimensionless temperature as $T = T_0(\tau T_* + 1)$, where $\tau = \Delta T/T_0$. By its own turn, density is assumed to vary with temperature through a first-order polynomial as

$$\rho = \rho_0[1 - \beta(T - T_0)] \tag{10}$$

where ρ_0 is the density for the reference temperature T_0 and β is the volumetric expansion coefficient. The problem is analyzed in its dimensionless form, and the governing variables are made dimensionless as

$$(x_*, y_*) \equiv (x, y)/H \tag{11}$$

$$(u_*, v_*) \equiv (u, v)/(\alpha_{\mathrm{m}}/H) \tag{12}$$

$$\psi_* \equiv \psi/(\rho_0\alpha_{\mathrm{m}}) \tag{13}$$

If the fluid is taken as an ideal gas then $\beta T = 1$, and

$$\beta_* = \frac{\beta}{\beta_0} = \frac{T_0}{T} = \frac{1}{\tau T_* + 1}; \quad \rho_* = \frac{\rho}{\rho_0} = \frac{1}{\tau T_* + 1} = \beta_* \tag{14}$$

In this work, an ideal gas means only that $\beta T = 1$, and not that a closed thermodynamic relationship exists between temperature and pressure. In other words, the fluid is taken as incompressible (density unaffected by pressure changes) but dilatable (density affected by temperature changes). For any fluid, the temperature is obtained from the energy conservation equation and pressure is obtained from (or as part of) the flow solution. In this case, if the fluid is taken as an ideal gas, the dimensionless version of Eq. (8) is

$$0 = \frac{\partial}{\partial x_*}\left(\frac{1}{\rho_*}\frac{\partial \psi_*}{\partial x_*}\right) + \frac{\partial}{\partial y_*}\left(\frac{1}{\rho_*}\frac{\partial \psi_*}{\partial y_*}\right) + Ra\rho_*^2\frac{\partial T_*}{\partial x_*} \tag{15}$$

where Ra is the Darcy-modified Rayleigh number

$$Ra \equiv \frac{g\beta_0\Delta TKH}{\nu\alpha_{\mathrm{m}}} \tag{16}$$

noting that $\beta_0 = 1/T_0$ in this case.

If, instead, the fluid cannot be taken as an ideal gas then

$$\beta_* = \frac{\beta}{\beta_0} = 1; \quad \rho_* = \frac{\rho}{\rho_0} = 1 - \beta_0\Delta TT_* \tag{17}$$

and the dimensionless version of Eq. (8) is

$$0 = \frac{\partial}{\partial x_*}\left(\frac{1}{\rho_*}\frac{\partial \psi_*}{\partial x_*}\right) + \frac{\partial}{\partial y_*}\left(\frac{1}{\rho_*}\frac{\partial \psi_*}{\partial y_*}\right) + Ra\frac{\partial T_*}{\partial x_*} \tag{18}$$

the Darcy-modified Rayleigh number being the same as defined in Eq. (16), but in this case β_0 is a constant value, independent of temperature.

If the Oberbeck–Bousinesq approximation is considered, which assumes that density is constant everywhere exception made to the buoyancy term [15], Eq. (18) applies but with the terms $1/\rho_* = 1$ within brackets (convective terms).

3.2. Energy conservation equation

Before considering the differential form of the energy conservation equation to be solved in order to obtain the temperature field, some previous explanation is given about the derivation of such an equation. This is made because a careful analysis of such an equation is needed, and its detailed derivation is not usual in the convection in porous media literature.

Following Bird et al. [22], the (total) energy conservation equation for the fluid, for a steady situation, can be stated in vector form as

$$0 = -\varepsilon\nabla \cdot \left(\frac{1}{2}\rho|\mathbf{V}|^2\right)\mathbf{V} - \varepsilon(\nabla \cdot \mathbf{q}_\mathrm{f}) - \varepsilon\nabla \cdot (\rho\hat{u})\mathbf{V} - \varepsilon\nabla p \cdot \mathbf{V} - \varepsilon p(\nabla \cdot \mathbf{V}) + \varepsilon\rho(\mathbf{V} \cdot \mathbf{g}) \quad (19)$$

where the dot means scalar product, $\hat{u}$ is the specific internal energy of the fluid and only the term that is not related with the fluid velocity, $\mathbf{q}_\mathrm{f}$, was referenced with subscript f. In this equation it is assumed that, when the Darcy Law is used to describe the flow field, there are no shear stresses in the fluid and thus there is no work of the shear stresses in the energy conservation equation for the fluid. This equation is general, provided that there is no work of shear stresses, and it applies both when buoyancy effects are present or not, with $\mathbf{g} = 0$ when the buoyancy effects are absent. The (total) energy conservation equation for the solid matrix, also for a steady situation, can be stated in vector form as

$$0 = -(1-\varepsilon)(\nabla \cdot \mathbf{q}_\mathrm{s}) \quad (20)$$

Eq. (19) can be rewritten using the Dupuit–Forchheimer relationship as

$$0 = -\nabla \cdot \left(\frac{1}{2\varepsilon}\rho|\mathbf{v}|^2\right)\mathbf{v} - \varepsilon(\nabla \cdot \mathbf{q}_\mathrm{f}) - \nabla \cdot (\rho\hat{u})\mathbf{v} - \nabla p \cdot \mathbf{v} - p(\nabla \cdot \mathbf{v}) + \rho(\mathbf{v} \cdot \mathbf{g}) \quad (21)$$

From Eq. (4) it can be shown that

$$\mathbf{v} \cdot \mathbf{v} = |\mathbf{v}|^2 = -\frac{K}{\mu}[\mathbf{v} \cdot \nabla p - \rho(\mathbf{v} \cdot \mathbf{g})] \quad (22)$$

and it is thus $-\mathbf{v} \cdot \nabla p + \rho(\mathbf{v} \cdot \mathbf{g}) = (\mu/K)|\mathbf{v}|^2$, which, when substituted in Eq. (21) leads to

$$0 = -\nabla \cdot \left(\frac{1}{2\varepsilon}\rho|\mathbf{v}|^2\right)\mathbf{v} - \varepsilon(\nabla \cdot \mathbf{q}_\mathrm{f}) - \nabla \cdot (\rho\hat{u})\mathbf{v} - p(\nabla \cdot \mathbf{v}) + \frac{\mu}{K}|\mathbf{v}|^2 \quad (23)$$

where the term $-p(\nabla \cdot \mathbf{v})$ can be identified as the reversible rate of internal energy increase by unit of volume, by compression [22], and it must be retained that the terms involving velocity refer to the fluid only.

When dealing with a clear fluid, the internal energy conservation equation (sometimes referred to as the thermal energy equation) is obtained by subtracting the mechanical energy equation (or kinetic energy equation) from the total energy equation [22]. In that case, no kinetic energy terms remain in the internal energy equation, because the inertial terms of the momentum equation give rise to a convective kinetic energy term in the kinetic energy equation, and the two kinetic energy terms cancel in the thermal energy equation. When the Darcy Law is used, a kinetic term remains in the energy equation, Eq. (23), which will be referred here as the thermal energy equation for the fluid.

This equation can be rewritten, using the concept of material derivative, as

$$\rho\frac{\mathrm{D}\hat{u}}{\mathrm{D}t} = -\nabla \cdot \left(\frac{1}{2\varepsilon}\rho|\mathbf{v}|^2\right)\mathbf{v} - \varepsilon(\nabla \cdot \mathbf{q}_\mathrm{f}) - p(\nabla \cdot \mathbf{v}) + \frac{\mu}{K}|\mathbf{v}|^2 \quad (24)$$

It is to be noted that, for a steady situation, the energy conservation equation can be written in this form, as the unsteady terms are effectively null (no energy accumulation in the fluid or in the solid matrix) and that the convective terms refer to the fluid phase only.

From the definition of specific enthalpy (for the fluid), $\hat{h} = \hat{u} + p/\rho$, it is

$$\frac{\mathrm{D}\hat{h}}{\mathrm{D}t} = \frac{\mathrm{D}\hat{u}}{\mathrm{D}t} + \frac{1}{\rho}\frac{\mathrm{D}p}{\mathrm{D}t} - \frac{p}{\rho^2}\frac{\mathrm{D}\rho}{\mathrm{D}t} \quad (25)$$

and invoking the mass conservation equation it is $(p/\rho)(\mathrm{D}\rho/\mathrm{D}t) = -p(\nabla \cdot \mathbf{v})$, and the energy conservation equation for the fluid becomes

$$\rho\frac{\mathrm{D}\hat{h}}{\mathrm{D}t} - \frac{\mathrm{D}p}{\mathrm{D}t} = -\nabla \cdot \left(\frac{1}{2\varepsilon}\rho|\mathbf{v}|^2\right)\mathbf{v} - \varepsilon(\nabla \cdot \mathbf{q}_\mathrm{f}) + \frac{\mu}{K}|\mathbf{v}|^2 \quad (26)$$

From thermodynamic principles [20] it can be written that

$$\rho\frac{\mathrm{D}\hat{h}}{\mathrm{D}t} = \rho c_P\frac{\mathrm{D}T}{\mathrm{D}t} + (1-\beta T)\frac{\mathrm{D}p}{\mathrm{D}t} \quad (27)$$

and Eq. (26) becomes

$$\rho c_P\frac{\mathrm{D}T}{\mathrm{D}t} - \beta T\frac{\mathrm{D}p}{\mathrm{D}t} = -\nabla \cdot \left(\frac{1}{2\varepsilon}\rho|\mathbf{v}|^2\right)\mathbf{v} - \varepsilon(\nabla \cdot \mathbf{q}_\mathrm{f}) + \frac{\mu}{K}|\mathbf{v}|^2 \quad (28)$$

The energy conservation equation for the fluid, Eq. (28), can be added to the energy conservation for the solid phase, Eq. (20), to give the thermal energy conservation equation for the steady fluid-saturated porous medium as

$$\rho c_P\frac{\mathrm{D}T}{\mathrm{D}t} - \beta T\frac{\mathrm{D}p}{\mathrm{D}t} = -\nabla \cdot \left(\frac{1}{2\varepsilon}\rho|\mathbf{v}|^2\right)\mathbf{v} - (\nabla \cdot \mathbf{q}_\mathrm{m}) + \frac{\mu}{K}|\mathbf{v}|^2 \quad (29)$$

where $\mathbf{q}_\mathrm{m} = \varepsilon\mathbf{q}_\mathrm{f} + (1-\varepsilon)\mathbf{q}_\mathrm{s}$.

If a quadratic drag term is considered, expressing the momentum equation as $\nabla p - \rho\mathbf{g} = -[(\mu/K) + c_F K^{-1/2}\rho|\mathbf{v}|]\mathbf{v}$, as proposed by Nield and Bejan [15], it can be shown that $-\mathbf{v}\cdot\nabla p + \rho(\mathbf{v}\cdot\mathbf{g}) = [(\mu/K) + c^F K^{-1/2}\rho|\mathbf{v}|]|\mathbf{v}|^2$. In this case, the viscous dissipation term entering in the energy conservation equation, as well as in the entropy generation equation, is not only $(\mu/K)|\mathbf{v}|^2$ but $[(\mu/K) + c_F K^{-1/2}\rho|\mathbf{v}|]|\mathbf{v}|^2$, and the quadratic drag term, even if it does not include the viscosity in an explicit way, effectively enters in these equations [16].

Eq. (29) can be rewritten extensively, in the conservative form, as

$$\frac{\partial}{\partial x}[\rho u c_P (T - T_0)] + \frac{\partial}{\partial y}[\rho v c_P (T - T_0)] = \frac{\partial}{\partial x}\left(k_m \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_m \frac{\partial T}{\partial y}\right) + \left\{\beta T\left(u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y}\right) + \frac{\mu}{K}|\mathbf{v}|^2 - \frac{1}{2\varepsilon}\left[\frac{\partial}{\partial x}\left(\rho u|\mathbf{v}|^2\right) + \frac{\partial}{\partial y}\left(\rho v|\mathbf{v}|^2\right)\right]\right\} \tag{30}$$

This equation can be worked using Eq. (5) to express the pressure gradient components, and made dimensionless to give

$$\frac{\partial}{\partial x_*}(\rho_* u_* T_*) + \frac{\partial}{\partial y_*}(\rho_* v_* T_*) = \frac{\partial^2 T_*}{\partial x_*^2} + \frac{\partial^2 T_*}{\partial y_*^2} - \frac{1}{2\varepsilon}Ec\left[\frac{\partial}{\partial x_*}(\rho_* u_* |\mathbf{v}_*|^2) + \frac{\partial}{\partial y_*}(\rho_* v_* |\mathbf{v}_*|^2)\right] - \beta_0 T_0 \beta_* \frac{EcPr}{Da}(\tau T_* + 1)\left[|\mathbf{v}_*|^2 + \frac{Ra}{\beta_0 \Delta T}\rho_* v_*\right] + \frac{EcPr}{Da}|\mathbf{v}_*|^2 \tag{31}$$

where

$$Ec \equiv \frac{(\alpha_m/H)^2}{c_P \Delta T} \tag{32}$$

$$Pr \equiv \nu/\alpha_m \tag{33}$$

$$Da \equiv \frac{K}{H^2} \tag{34}$$

When dealing with a domain filled with a clear fluid, thermal energy conservation equation, obtained as the total energy conservation equation subtracted of the mechanical energy conservation equation (kinetic energy equation) leads to an equation that does not include a kinetic energy term. In the case of a fluid-saturated porous medium, the energy conservation equation includes a convective term of the kinetic energy. However, as the Darcy number is usually a small value, and the Prandtl number is close to the unity, the kinetic energy term is some orders of magnitude lower than the other terms, and it can be neglected in the energy conservation equation, what was confirmed from numerical experiments in this work. This point forward, in this work, this term of kinetic energy will not be considered in the thermal energy conservation equation.

The two last terms in Eq. (31) are identified as the work of pressure forces and as the viscous dissipation, respectively. Application of the First Law of Thermodynamics to the overall enclosure gives

$$\frac{(-\dot{Q}_H - \dot{Q}_C)}{k_m \Delta T} + \int_{V_*} -\beta_0 T_0 \beta_* \frac{EcPr}{Da}(\tau T_* + 1) \times \left[|\mathbf{v}_*|^2 + \frac{Ra}{\beta_0 \Delta T}\rho_* v_*\right] dV_* + \int_{V_*} \frac{EcPr}{Da}|\mathbf{v}_*|^2 dV_* = 0 \tag{35}$$

where the integrals extend to the overall domain of the enclosure. The volume integrals of the divergence of the dimensionless heat flux were transformed into surface integrals using the Gauss' Theorem, and they give the dimensionless heat flows (the Nusselt numbers) entering and leaving the domain [1]. As $\dot{Q}_H + \dot{Q}_C = 0$, it is

$$\int_{V_*} -\beta_0 T_0 \beta_* (\tau T_* + 1)\left[|\mathbf{v}_*|^2 + \frac{Ra}{\beta_0 \Delta T}\rho_* v_*\right] dV_* + \int_{V_*} |\mathbf{v}_*|^2 dV_* = 0 \tag{36}$$

Locally, the terms corresponding to the work of pressure forces and to the viscous dissipation can be different. However, the integral of these two energy terms extended to the overall enclosure must cancel as given by Eq. (36).

If the fluid is taken as an ideal gas it is $\beta_0 T_0 \beta_* (\tau T_* + 1) = 1$ and Eq. (31) becomes

$$\frac{\partial}{\partial x_*}(\rho_* u_* T_*) + \frac{\partial}{\partial y_*}(\rho_* v_* T_*) = \frac{\partial^2 T_*}{\partial x_*^2} + \frac{\partial^2 T_*}{\partial y_*^2} - \frac{EcPr}{Da}\frac{Ra}{\beta_0 \Delta T}\rho_* v_* \tag{37}$$

ρ_* being evaluated as $\rho_* = 1/(\tau T_* + 1)$, as given by Eq. (14). From this result it is thus concluded that if the flow is essentially horizontal ($v_* = 0$), no source or sink exists in the energy conservation equation. This seems to be a strange result, as the viscous dissipation is present ever the fluid moves, no matter what the direction of the flow. It must be retained that the flow close to the horizontal walls of the enclosure is due only to the pressure gradient (null buoyancy effect), the variations in pressure being associated with the work of pressure forces that, locally, balance the viscous dissipation. Going to Eq. (30) it can be concluded that it is $u > 0$ and $(\partial p/\partial x) < 0$ close to the upper horizontal wall and $u < 0$ and $(\partial p/\partial x) > 0$ close to the lower horizontal wall. In both cases it is $u(\partial p/\partial x) < 0$, a term that, locally, balances the viscous dissipation term, $(\mu/K)|\mathbf{v}|^2$. Result expressed by Eq. (36) becomes, in this case

$$\int_{V_*} \rho_* v_* dV_* = 0 \tag{38}$$

If the fluid cannot be taken as an ideal gas then $\beta_* = 1$ and Eq. (31) becomes

$$\frac{\partial}{\partial x_*}(\rho_* u_* T_*) + \frac{\partial}{\partial y_*}(\rho_* v_* T_*) = \frac{\partial^2 T_*}{\partial x_*^2} + \frac{\partial^2 T_*}{\partial y_*^2} + \frac{EcPr}{Da}\left[-\beta_0 T_0(\tau T_* + 1)\left(|\mathbf{v}_*|^2 + \frac{Ra}{\beta_0 \Delta T}\rho_* v_*\right) + |\mathbf{v}_*|^2\right] \tag{39}$$

ρ_* being evaluated as $\rho_* = 1 - \beta_0 \Delta T T_*$, as given by Eq. (17). In this case, result expressed by Eq. (36) becomes

$$\int_{V_*}\left[-\beta_0 T_0(\tau T_* + 1)\left(|\mathbf{v}_*|^2 + \frac{Ra}{\beta_0 \Delta T}\rho_* v_*\right) + |\mathbf{v}_*|^2\right]\mathrm{d}V_* = 0 \tag{40}$$

If the Oberbeck–Bousinesq approximation is considered then $\rho_* = 1$ in Eqs. (39) and (40) (constant density everywhere except on the buoyancy term in the vertical momentum equation).

Like as referred when dealing with an enclosure filled with a clear fluid, also in the case of the enclosure filled with a fluid-saturated porous medium, special care must be taken when considering the viscous dissipation effects in the energy conservation equation in order to have heat transfer results consistent with the First Law of Thermodynamics, noting that: (i) Result $|\dot{Q}_C| = |\dot{Q}_H|$, or $Nu_C = Nu_H$, is the unique respecting the First Law of Thermodynamics. (ii) The fluid and temperature fields must be evaluated from the correct (and complete) formulation of the energy conservation equation, including all the relevant sources and sinks. (iii) Integration methods used must be consistent with the result given by Eq. (36); and (iv) Assessment of using the Oberbeck–Boussinesq approximation must be related not only with the difference on the involved thermal levels [23], but also on the verification of Eq. (40), noting that the real influence of the density variations spreads over the involved equations.

3.3. Entropy generation analysis

The volumetric entropy generation rate includes the contributions of heat conduction and of viscous dissipation, being expressed as

$$\dot{S}'''_{\mathrm{gen}} = \frac{k_m}{T^2}\left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right] + \frac{\mu}{KT}|\mathbf{v}_*|^2 \tag{41}$$

In what concerns the total entropy generation in the overall enclosure, as given by Eq. (2), it can be expressed as

$$\dot{S}_{\mathrm{gen}} = \dot{Q}\left(\frac{1}{T_C} - \frac{1}{T_H}\right) = Nu\dot{Q}_{CD}\left(\frac{1}{T_C} - \frac{1}{T_H}\right) = Nuk_m\left(\frac{\tau^2}{1+\tau}\right) \tag{42}$$

where the overall Nusselt number is defined as

$$Nu \equiv \frac{\dot{Q}}{\dot{Q}_{CD}} = \frac{\dot{Q}}{k_m(H \times 1)(T_H - T_C)/H} \tag{43}$$

From Eq. (1) it must be $Nu_C = Nu_H = Nu$.

Making the overall entropy generation rate dimensionless as

$$\dot{S}_{\mathrm{gen},*} = \frac{\dot{S}_{\mathrm{gen}}}{k_m(\Delta T/T_0)} = Nu\left(\frac{\tau}{1+\tau}\right) \tag{44}$$

the dimensionless volumetric entropy generation rate is

$$\dot{S}'''_{\mathrm{gen},*} = \underbrace{\frac{\tau}{(\tau T_* + 1)^2}\left[\left(\frac{\partial T_*}{\partial x_*}\right)^2 + \left(\frac{\partial T_*}{\partial y_*}\right)^2\right]}_{\dot{S}'''_{\mathrm{gen},*,CD}} + \underbrace{\frac{EcPr}{Da}\frac{1}{(\tau T_* + 1)}|\mathbf{v}_*|^2}_{\dot{S}'''_{\mathrm{gen},*,D}} \tag{45}$$

where the first term refers to heat conduction and the second term refers to viscous dissipation, and it is $\dot{S}_{\mathrm{gen},*} = \int_{V_*}\dot{S}'''_{\mathrm{gen},*}\mathrm{d}V_*$. Study of the entropy generation is important for a better understanding of the convection heat transfer processes, as illustrated by Mahmud and Fraser [24] for some fundamental convective heat transfer problems.

3.4. Note about the numerical values of the dimensionless parameters

When solving the problem in its dimensionless form, care must be taken in what concerns the numerical values assumed by the dimensionless governing parameters. For air, water or other common fluids at room temperature, it can be seen that practical laminar situations lead to $Ra \sim 10 - 10^3$ and $Ec < \sim 10^{-9}$, parameters for which the dissipation effects are not 'visible'. In an attempt to increase the importance of the viscous dissipation effects, it is tempting to manipulate the values of the dimensionless parameters. From Eqs. (16), (32) and (34) it can be obtained that

$$H = Ra\left(\frac{EcPr}{Da}\right)\left(\frac{c_P}{g\beta_0}\right) \tag{46}$$

Taking $Ra = 10^3$ and $(EcPr/Da) = 10^{-3}$, for fluids at room temperature it is: $H = 3.1 \times 10^4$ m for air, $H = 1.5 \times 10^6$ m for water, and $H = 2.8 \times 10^5$ m for oil, that is, non-realistic values for the side length of the differentially heated square enclosure filled with a fluid-saturated porous medium.

4. Numerical modeling and illustrative results

The natural convection problem in the square enclosure filled with a fluid-saturated porous medium, with side length H, was solved in its dimensionless form as given by Eqs. (15) and (37) if the saturating fluid is taken as an ideal gas, or as given by Eqs. (18) and (39) if the fluid cannot be taken as an ideal gas. A control volume finite element method [25] was used, with a 101×101 non-uniform mesh, expanding from the walls to the center with a geometric expansion factor equal to 1.065. The numerical solution was obtained when addition (extended to all the nodes of the domain) of the absolute values of the residuals of each of the dimensionless equations were lower than 10^{-8}.

Results were obtained for different conditions as detailed in Table 1, and maintaining constant the parameters $Pr = 1$, $\Delta T = 10$ K, $T_0 = 300$ K and $\beta_0 = 1/T_0$ K^{-1}. As referred before, in Section 3, in this work ideal gas means only that $\beta T = 1$, and not that a closed thermodynamic relationship exists between temperature and pressure. For any fluid, temperature is obtained from the energy conservation equation and pressure is unnecessary as the flow field is extracted from the (conservative) streamfunction field. Viscous dissipation effects were considered in all the analyzed cases.

From the results in the row 6 of Table 1 it is clearly observed that erroneous results are obtained if only the viscous dissipation term is taken into account in the energy conservation equation. In this case, heat output is nearly 1.5 times the heat input, and the enclosure behaves like a heat multiplier, violating the First Law of Thermodynamics. Stronger heat multipliers of this kind were presented by Saeid and Pop [8] and by Rees [9]. Entropy generation is mainly due to viscous dissipation, but Eq. (2) could not apply as the heat transfer results are not in accordance with Eq. (1). The same conclusions apply equally to any situation for which only the viscous dissipation term is taken into account (row 2 of Table 1).

It can be also observed that strong changes exist in the heat transfer performance of the enclosure, given by the Nusselt number, when the work of pressure forces is considered or not, as obtained comparing results on rows 1 and 2, and 5 and 6 of Table 1. Thermal performance of the enclosure is considerably lower when the work of pressure forces is not taken into account. It must be retained, however, that results indicated in rows 2 and 6 of Table 1 are incorrect in what concerns the First Law of Thermodynamics when applied to the overall enclosure.

If the fluid can be taken as an ideal gas, as for the situations in rows 4 and 8 of Table 1, very good results are obtained in what concerns the verification of the First Law of Thermodynamics, the Nusselt numbers evaluated at the hot and cold vertical walls being very close to each other. Also in these cases it is $D_* = -W_*$, within the limits of the numerical approximations. In what concerns the entropy generation analysis it is $S_* = 0.0580 + 0.0443 = 0.1023$ for the row 4 of Table 1, and $S_* = 0.2166 + 0.2576 = 0.4742$ for the row 8 of Table 1, where the first values refer to the entropy generation due to heat conduction and the second values refer to viscous dissipation. Using Eq. (44) to evaluate the overall dimensionless entropy generation rate it is obtained that $(\dot{S}_{\mathrm{gen},*}) = Nu\tau(\tau+1)^{-1} = 0.1029$ for the situation listed in row 4 of Table 1, and that $(\dot{S}_{\mathrm{gen},*}) = Nu\tau(\tau+1)^{-1} = 0.4734$ for the situation listed in row 8 of Table 1. These overall results for the entropy generation are in good agreement (taking into account the introduced approximations) with the ones evaluated from the separated heat conduction and viscous dissipation contributions.

If the fluid could not be taken as an ideal gas, as for the situations listed in rows 1, 3, 5 and 7 of Table 1, it can be seen that slightly better results (regarding the verification of the First Law of Thermodynamics) are obtained when the Oberbeck–Boussinesq approximation is not used. It must be noted that it is not obtained the expected complete verification of the First Law of Thermodynamics even if the Oberbeck–Boussinesq approximation is not used, and that consideration of the Oberbeck–Boussinesq approximation leads to slightly higher values of the Nusselt numbers at the vertical isothermal walls. In relative terms, using the parameter $(Nu_{\mathrm{H}} - Nu_{\mathrm{C}})/Nu_{\mathrm{H}}$, it is obtained that the deviation from the First Law of Thermodynamics is only of 1.5% for the situation in the first row of Table 1, 1.4% for the situation in row 3, 1.8% for the situation in row 5, and 1.8% for the situation in row 7 of Table 1. It must be retained also that, contrarily to what happens when only the viscous dissipation is considered in the energy conservation equation, heat entering the enclosure is (slightly) higher than heat leaving the enclosure, and it behaves like a small heat sink. Further research needs to be made in order to find the source of this slight inconsistency regarding the First Law of Thermodynamics. It must be referred, however, that results are consistent with Eq. (35) even if they are not completely consistent with Eq. (36). For example, for the situation corresponding to the row 5 of Table 1, the dimensionless version of Eq. (35) gives

Table 1
Different conditions and terms in the energy conservation equation, Nusselt numbers at the hot and cold vertical walls, and results for the dimensionless overall viscous dissipation (always considered), D_*, dimensionless overall work of pressure forces, W_*, and dimensionless entropy generation, S_*

$\frac{EcPr}{Da}$	Ra	Oberbeck–Boussinesq approximation	Ideal gas	Work of pressure forces	Nu_{H}	Nu_{C}	D_*	$-W_*$	S_*
0.001	50	No	No	Yes	3.291	3.243	0.0473	0.0950	0.1056
0.001	50	No	No	No	1.932	1.999	0.0670	0.0000	0.1282
0.001	50	Yes	No	Yes	3.322	3.274	0.0470	0.0948	0.1058
0.001	50	No	Yes	Yes	3.189	3.188	0.0450	0.0458	0.1023
0.005	100	No	No	Yes	15.21	14.93	0.273	0.551	0.4913
0.005	100	No	No	No	2.531	3.640	1.110	0.000	1.1891
0.005	100	Yes	No	Yes	15.34	15.07	0.271	0.5422	0.4913
0.005	100	No	Yes	Yes	14.68	14.67	0.262	0.279	0.4742

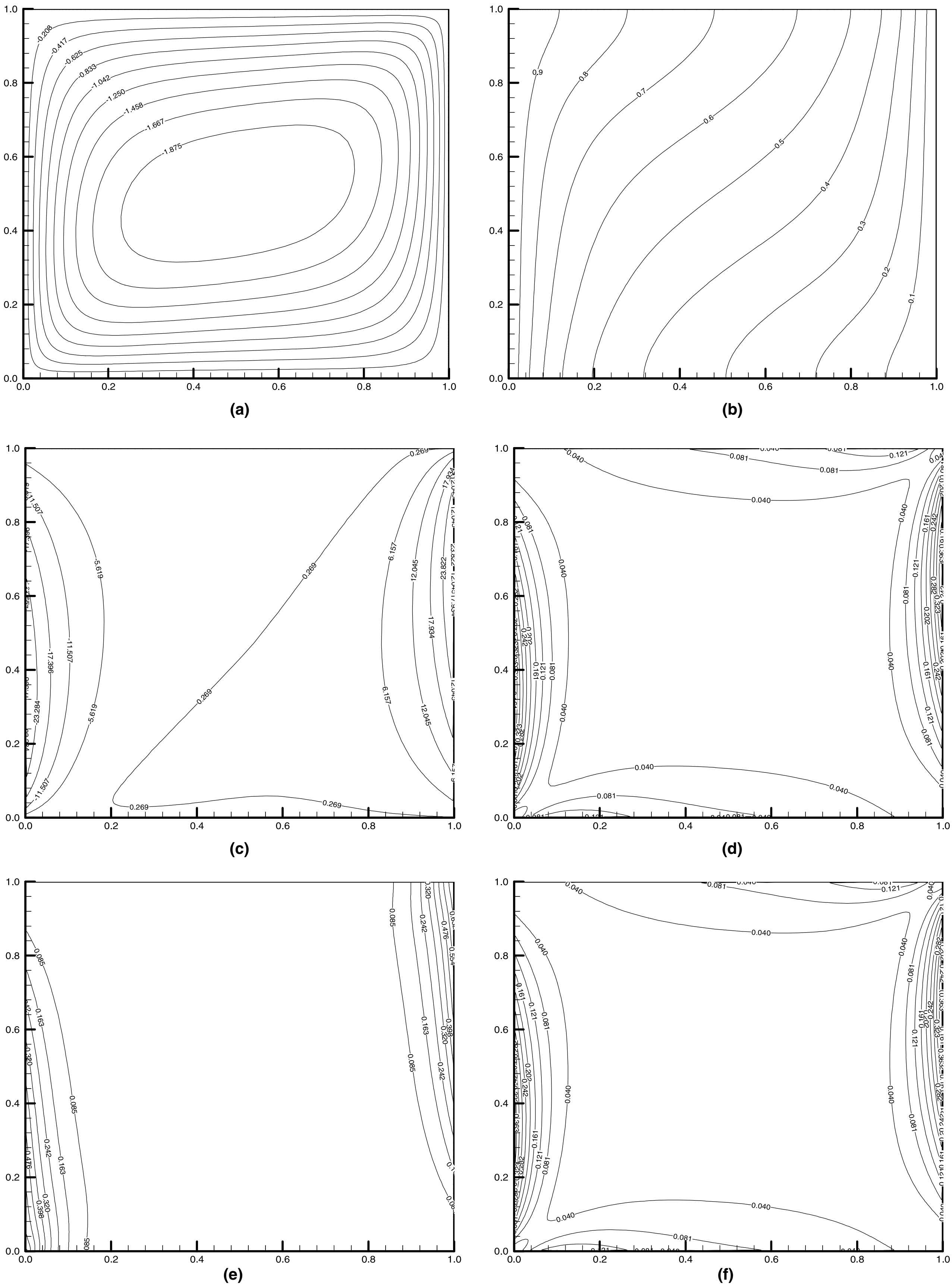

Fig. 2. Contour plots of dimensionless variables for the situation listed in row 4 of Table 1: (a) (conservative) streamfunction; (b) temperature; (c) local work rate of pressure forces; (d) local viscous dissipation rate; (e) local entropy generation rate associated with heat transfer; and (f) local entropy generation rate associated with viscous dissipation.

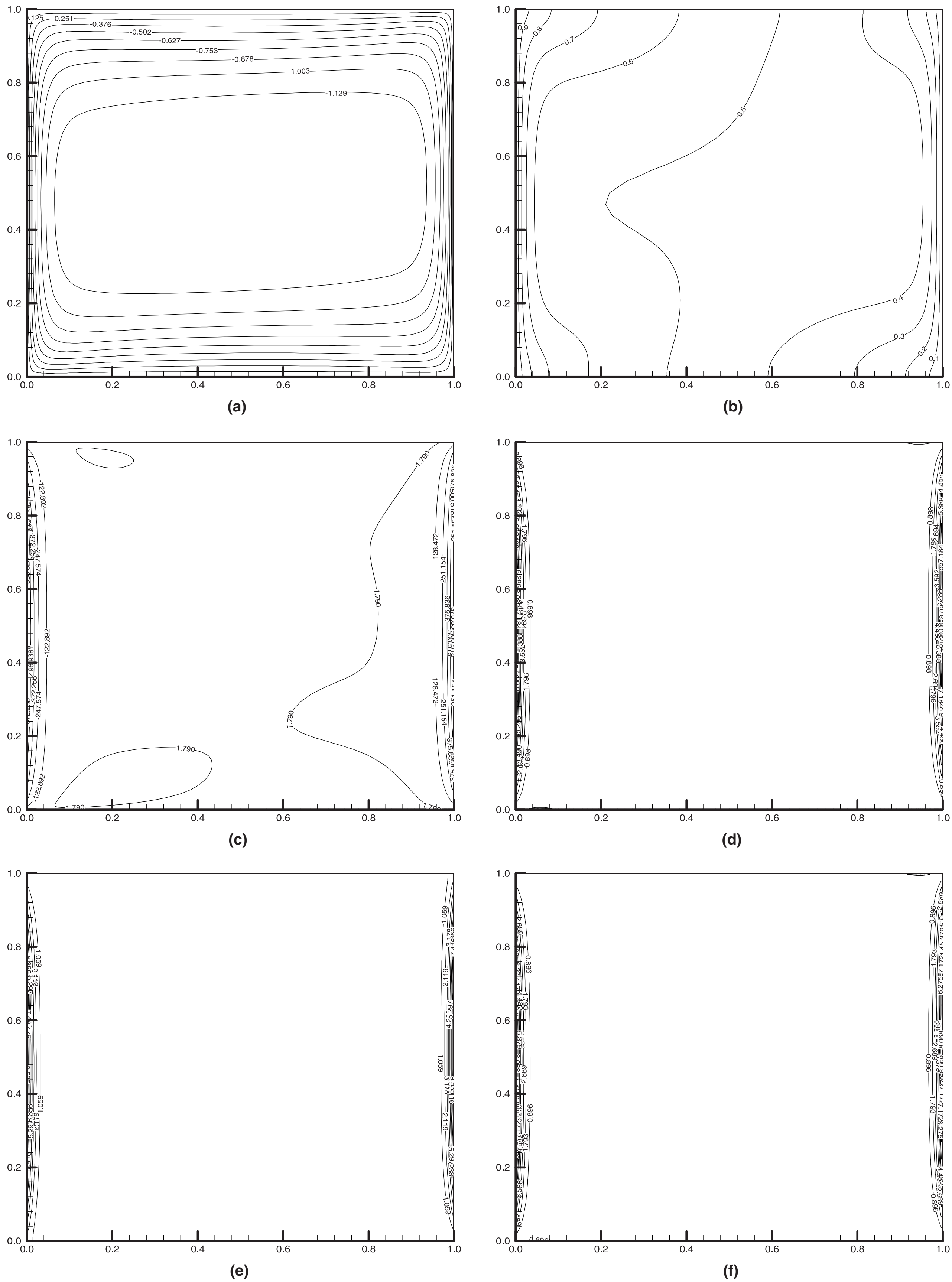

Fig. 3. Base legend as for Fig. 2, now referring to the situation listed in row 8 of Table 1.

$-15.21 + 14.93 - 0.273 + 0.551 = -0.002$, and Eq. (35) is verified within the approximation limits.

In what concerns the (conservative) streamfunction, temperature, work of pressure forces, viscous dissipation, entropy generation associated with heat conduction and with viscous dissipation, some results are presented considering the fluid as an ideal gas for the conditions listed in rows 4 and 8 of Table 1.

For the situation in row 4 of Table 1, streamfuncion (Fig. 2a) and temperature (Fig. 2b) contour plots are not very different from the ones typical in low intensity natural convection in enclosures filed with fluid-saturated porous media. In what concerns the local work of pressure forces, in Fig. 2c, it is important close to the vertical walls and not at the center of the enclosure as well as close to the insulated horizontal walls. Local heating due to viscous dissipation is presented in Fig. 2d, and it is observed that it is relevant close to all the enclosure walls. It is to be retained that the absolute value of the work of pressure forces is considerably higher than that of heat release by viscous dissipation. However, viscous dissipation is positive everywhere, the work of pressure forces being negative close to the hot vertical wall and positive close to the cold vertical wall. Entropy generation rate due to heat conduction, presented in Fig. 2e, is higher where conduction heat transfer is more intense, that is, close to the vertical isothermal walls and, in particular, close to the lower half of the hot wall and close to the upper half of the cold wall. Entropy generation rate associated with viscous dissipation closely follows the heat generation by viscous dissipation, as obtained comparing Fig. 2f and d. Similar results for the entropy generation rate were obtained also by Baytas [17], even if the formulation of the problem presents some differences as pointed in Section 1.

Considerable changes are observed when convection and viscous dissipation increase in intensity (higher Rayleigh and Eckert numbers, respectively), here corresponding to the conditions listed in row 8 of Table 1. Streamlines of Fig. 3a are very concentrated near the vertical walls, thus indicating the high intensity of fluid flow there. The central part of the enclosure corresponds nearly to motionless fluid. Contour plots of temperature in Fig. 3b indicate that there are strong thermal gradients close to the vertical walls, and thus intense heat transfer there, in complete agreement with the overall results for the Nusselt number listed in Table 1. In what concerns local work of pressure forces, in Fig. 3c, local heat generation associated with viscous dissipation, in Fig. 3d, local entropy generation associated with heat conduction, in Fig. 3e, and local entropy generation associated with viscous dissipation, in Fig. 3f, these can be seen as similar to the ones in Fig. 2, but now with the important changes very concentrated close to the vertical walls. In this case, the relevant phenomena and processes occur close to the vertical walls of the enclosure, and its central part consists essentially of motionless isothermal fluid, at a temperature $(T_H + T_C)/2$.

In order to have a better picture about the involved numerical values of heat transfer, viscous dissipation and work of pressure forces, some more attention is given to the situation corresponding to the last row of Table 1. If the fluid that saturates the porous medium is air at room temperature, with $Pr = 0.73 \approx 1$, for $EcPr/Da = 0.001$ and $Ra = 100$ Eq. (46) gives that $H \approx 3000$ m, a considerably high value as pointed in Section 3 in the note about the numerical values of the dimensionless parameters. Evaluating the thermal conductivity of the saturated porous medium as $k_m = k_f^{\varepsilon} k_s^{(1-\varepsilon)}$ [15], where ε is porosity, it can be taken $k_m \approx 0.1$ W m^{-1} K^{-1}. Values of Table 1 were obtained for a temperature difference $\Delta T = 10$ K between the vertical walls of the enclosure, and heat transfer across the enclosure can be evaluated from last row of Table 1 as $\dot{Q}_H = Nu_H k_m \Delta T = 14.68$ W (by unit depth of the 2D enclosure). This is a small heat transfer rate, but it is to be retained that the enclosure has a low overall thermal conductivity and that distance separating the hot and cold walls has 3 km thickness. In what concerns the dimensional values of the viscous dissipation and of the work of pressure forces, they can be evaluated from last row of Table 1 as $\dot{D} = \dot{D}_* k_m \Delta T = 0.262$ W and $\dot{W} = \dot{W}_* k_m \Delta T = -0.279$ W, respectively (by unit depth of the 2D enclosure). These are only small energy interactions that can be neglected in many situations of practical interest. If, however, higher values were considered for the dimensionless parameters, the considered enclosure was more non-realistic, and the energy interactions were greater.

5. Conclusions

Energy conservation in the natural convection heat transfer problems in enclosures filled with fluid-saturated porous media, including viscous dissipation effects, must be carefully formulated, in order to be consistent with the First Law of Thermodynamics. If the viscous dissipation term is included in the energy conservation equation, also the work of pressure forces needs to be considered. Viscous dissipation results from fluid motion, which is due to the expansion–contraction cycle experienced by the fluid in the enclosure, and the work of pressure forces cannot be neglected. If only the viscous dissipation term is taken into account, the fluid-saturated porous medium behaves like a heat multiplier, for which the energy conservation principle is not respected.

Locally, the work of pressure forces can be different from the energy released by viscous dissipation, but one relevant result is that the integral of these terms, extended to the overall fluid-saturated porous domain, must cancel. This result comes from the direct application of the First Law of Thermodynamics to the overall enclosure, and it can be used to assess if the energy conservation principle is being respected or not. This can be very important when analyzing the relevance of the different terms of elaborated fluid flow models to describe the fluid flow in fluid-saturated porous media, like a Brinkman viscous term, a quadratic drag term, and inertial terms, or even the complete model given by the Brinkman–Forchheimer equations. This same result was used to interpret the obtained results for the Nusselt numbers, as well as to take some considerations about the ideal gas model or the use of the Oberbeck–Boussinesq approximation.

The main result of this work applies also to mixed convection in fluid-saturated porous media, taking into consideration that part of the viscous dissipation comes from the forced flow (mechanical energy input) and part comes from the work of pressure forces (associated with the expansion–contraction cycle experienced by the fluid). Only the complete (and correct) energy conservation equation can give the correct results in what concerns the flow and temperature fields, and the heat transfer performance of the fluid-saturated porous medium.

Increasing importance is being given to the thermodynamic analysis of the natural convection problems in enclosures, and a contribution is given by this work when the enclosure is filled with a fluid-saturated porous medium.

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